HYPER VERSION

Received: *June 4, 2001,* Accepted: *August 2, 2001*

UV/IR connection, a matrix perspective*

Yaron Kinar and Jacob Sonnenschein

Raymond and Beverly Sackler Faculty of Exact Sciences
School of Physics and Astronomy
Tel-Aviv University, Ramat Aviv, 69978 Israel
E-mail: yaronki@post.tau.ac.il, cobi@post.tau.ac.il

Gilad Lifschytz

Department of Mathematics and Physics
University of Haifa at Oranim, Qiryat-Tivon, Israel
E-mail: giladl@research.haifa.ac.il

Abstract: We show that the matrix formulation of non-commutative field theories is equivalent, in the continuum, to a formulation in a mixed configuration-momentum space. In this formulation, the non-locality of the interactions, that leads to the IR/UV mixing, becomes transparent. We clarify the relation between long range effects (and IR divergences) and the non-planarity of the corresponding Feynman diagrams.

Keywords: Non-Commutative Geometry, M(atrix) Theories.

*The work of Y.K and J.S. is supported in part by the US-Israel Binational Science Foundation, by GIF — the German-Israeli Foundation for Scientific Research, and by the Israel Science Foundation.

Contents

1. Introduction

While much work has been devoted to the understanding of field theories on non-commutative spaces (see [1]), in particular after those theories were related to string theory ([2]–[7]), some issues remain unclear. An important example of such an issue is the so called "IR/UV mixing" [8]. This peculiar behaviour of field theories on non-commutative spaces is manifested through the existence of a class of non-planar diagrams which are divergent for vanishing external momenta. The divergence originates from the phase of these non-planar diagrams that depends on both the external and internal momenta. The integration of the latter in loops results in UV divergences which are the source of the "IR divergences" of the non-planar diagrams.

The UV/IR mixing was further discussed both for scalar field theories ([9]–[17]) and for gauge theories which can be associated with the open string perspective ([18]–[24]). The immediate question of the effect of this mixing on the renormalizability of these theories was also intensively considered ([25]–[33]).

In this note we would like to further investigate the origin of the IR divergences and clarify their relations to non-commutativity, non-locality of the interaction, and non-planarity of the relevant diagrams.

We use the known mapping between a non-commutative field theory and a matrix theory. We make the observation that the formulation in terms of $N \times N$ matrices corresponds, in the continuum picture, to expressing the fields as functions of coordinates of a mixed configuration-momentum space. By using matrices of finite size we regularize both the UV and the IR divergences, since the maximal value of the momentum and the area are proportional to $\sqrt{N}$ and N respectively. We show, in the mixed configuration-momentum space, that the interactions of scalar field theories defined on non-commutative spaces are non-local. The non-locality renders the interactions into long range ones even for massive fields. For space with infinite volume, these long range interactions are behind the IR divergences associated with certain non-planar diagrams. We show that the effect of the non local interactions is to bring the interaction points closer to each other. This behaviour is the source of the "mixing" of the IR divergences with the contributions of high momentum that leads usually to UV divergences. We explain why this effect exists (and is natural) only for certain non-planar diagrams in the matrix theory.

In terms of the dependence on the cutoff N, we show that the UV divergent planar diagrams have the same N dependence as IR divergent non-planar diagrams of the same loop order. We demonstrate this behaviour for the case of the two point function. Each planar diagram in this case has a higher N dependence than that of a corresponding non-planar one due to the summation of all possible indices in closed internal loops. However, this factor is compensated by the multiplicity of non-planar diagrams. Thus, even in the $\theta \to \infty$ limit some non-planar diagrams contribute.

The paper is organized as follows. In section 2 we describe the basic setup, namely, the operator formulation of the scalar field theory on non-commutative geometry. Anticipating the structure of the matrix description, we rewrite in section 3 the action in a mixed configuration-momentum space representation. In section 4 we determine the propagator in the operator (matrix) framework, and show, using the 'shift' and 'clock' matrices, that it indeed corresponds to the mixed description. We state in section 5 the Feynman rules in the latter description as well as in the matrix formulation. Section 6 is devoted to the the origin of the IR divergences. In particular, it is shown that the noncommutativity introduces long range interactions, and explained why the IR divergences are associated with non-planar diagrams. We determine the dependence of the various diagrams on N. An example is described in the appendix. It includes the leading order corrections to the ϕ^3 two point function from both planar and non-planar diagram and determines the corresponding phases.

2. The basic setup

We start with a brief description of the basic setup of the generic theory we study, noting our conventions in passing. Consider the non-commutative ϕ^K field theory

JHEP08(2001)001

on a $(d+1)$-dimensional plane, whose euclidean action is

$$S = \int dt d^d x \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} M^2 \phi^2 + \lambda \phi^K \right), \tag{2.1}$$

where the products are Moyal $*$-products

$$(\phi_1 * \phi_2)(x) = \exp\left(\frac{i}{2} \theta^{\mu\nu} \partial_\mu^y \partial_\nu^z \right) \phi_1(y) \phi_2(z)|_{y=z=x} \,. \tag{2.2}$$

In particular

$$e^{ikx} * e^{ipx} = \exp\left(-\frac{i}{2} k \times p \right) e^{i(p+k)x} \tag{2.3}$$

or

$$e^{ikx} * e^{ipx} = \exp\left(-ik \times p\right) e^{ipx} * e^{ikx} \,, \tag{2.4}$$

where

$$k \times p \equiv k_\mu \theta^{\mu\nu} p_\nu \,. \tag{2.5}$$

We Fourier transform $\phi(x)$

$$\tilde{\phi}(k) = \frac{1}{\sqrt{(2\pi)^d}} \int d^d x e^{-ikx} \phi(x) \tag{2.6}$$

and represent the Moyal algebra by the operators $\hat{A}(\hat{x})$ defined as (see for instance [35, 36, 37])

$$\hat{A}(\hat{x}) = \frac{1}{\sqrt{(2\pi)^d}} \int d^d k e^{ik\hat{x}} \tilde{\phi}(k) \,. \tag{2.7}$$

where $\hat{x}^\mu$ are operators satisfying the commutation relations

$$[\hat{x}^\mu, \hat{x}^\nu] = -i\theta^{\mu\nu} \,. \tag{2.8}$$

We now formulate the non-commutative field theory using the operators formalism, according to the mapping

$$\begin{aligned} \phi_1 * \phi_2 &\to \hat{A} \cdot \hat{B} \\ \frac{1}{2\pi \mathrm{Pf}(\theta)} \int d^d x &\to \mathrm{Tr} \\ i\theta^{\mu\nu} \partial_\nu &\to [\hat{x}^\mu, \cdot] \,. \end{aligned} \tag{2.9}$$

For simplicity, we assume there are only two non-commutative coordinates, $rank(\theta) = 2$, $[\hat{x}^1, \hat{x}^2] = -i\theta$, and ignore all the other (commutative) coordinates. We define the exponents

$$\hat{U} \equiv \exp\left(-i\hat{x}^1\right); \qquad \hat{V} \equiv \exp\left(-i\hat{x}^2\right) \tag{2.10}$$

satisfying

$$\hat{U}\hat{V} = e^{-i\theta}\hat{V}\hat{U}\,. \tag{2.11}$$

Using these definitions the Fourier transform takes the following form

$$\begin{aligned}\tilde{\hat{A}}(p,q) &= \mathrm{Tr}\left(e^{-i(p\hat{x}^1+q\hat{x}^2)}\hat{A}\right)\\ &= \mathrm{Tr}\left(\exp\left(\frac{i}{2}\theta pq\right)e^{-ip\hat{x}^1}e^{-iq\hat{x}^2}\hat{A}\right)\\ &\equiv \mathrm{Tr}\left(:\hat{U}^p\hat{V}^q:\hat{A}\right)\end{aligned} \tag{2.12}$$

and the action

$$\int dt\,\mathrm{Tr}\left(\theta^{-2}[\hat{x}^\mu,\hat{A}]^2 + \frac{1}{2}M^2\hat{A}^2 + \lambda\hat{A}^K\right). \tag{2.13}$$

3. Mixed configuration-momentum space

The interesting relation between momentum and size in non-commutative field theories ([34]), guides us to consider the mixed configuration-momentum space. Working in two space dimensions x and y, one can Fourier transform the action only in the x-direction. The kinetic term becomes

$$S_{kin} = \frac{1}{2}\int dp_x dy\phi(\tilde{p_x},y)\left(p_x^2-\partial_y^2\right)\phi(\tilde{p_x},y)\,. \tag{3.1}$$

The notation is such that from now on, we use ϕ for the field in all representations, and indicate momentum-space coordinates by a tilde — $\tilde{p}$ and configuration space coordinate by a non-tilde variable — y. The mass term remains the same while the interaction becomes (considering ϕ^3 as a simple example) —

$$\begin{aligned}S_{\rm int} &= \int dxdy\phi(x,y)*\phi(x,y)*\phi(x,y)\\ &\sim \int dxdydp_1dp_2dp_3(e^{ip_1x}\phi(\tilde{p_1},y))*(e^{ip_2x}\phi(\tilde{p_2},y))*(e^{ip_3x}\phi(\tilde{p_3},y))\\ &= \int dydp_1dp_2\phi\left(\tilde{p_1},y-\frac{\theta p_1}{2}\right)\phi\left(\tilde{p_2},y-\theta p_1-\frac{\theta p_2}{2}\right)\phi\left(-\tilde{p_1}-\tilde{p_2},y-\frac{\theta p_1}{2}-\frac{\theta p_2}{2}\right).\end{aligned}$$

We can further change our notations and write

$$\begin{aligned}A(z,w) &\equiv \phi\left(\frac{1}{\theta}(\tilde{z}-\tilde{w}),\frac{1}{2}(z+w)\right)\\ \phi(\tilde{p},x) &= A\left(x+\frac{\theta p}{2},x-\frac{\theta p}{2}\right)\end{aligned} \tag{3.2}$$

and the interaction term becomes

$$\begin{aligned}S_{\rm int} &= \int dydp_1dp_2A(y,y-\theta p_1)A(y-\theta p_1,y-\theta p_1-\theta p_2)A(y-\theta p_1-\theta p_2,y)\\ &= \mathrm{Tr}\,A^3\,,\end{aligned}$$

where the last line can be considered formally or via discretization. From now on we will put $\theta = 1$ in our expressions (unless stated otherwise), it can be easily inserted back to restore the dimensionality. The last expression clearly shows that the action in the mixed configuration-momentum space has a matrix structure. In the next section we will show that the familiar 'clock' and 'shift' representation of the operators $\hat{U}$ and $\hat{V}$ indeed leads to this mixed space. We will further discuss the non-locality of the interaction term in section 6.

4. Operator (matrix) space

In order to consider the perturbative behaviour of the theory in the operator representation, the first step we take is to regularize any divergence by considering a finite dimensional Hilbert space. In other words, the various operators are now $N \times N$ matrices. Eventually, we will let N go to infinity. The degrees of freedom are the elements of the matrix $\hat{A}$.

It is well known that there is no finite dimensional matrix representation for the non-commuting operators $\hat{x}^1$ and $\hat{x}^2$. However, any $N \times N$ matrix can be expanded in a finite power series in two unitary matrices which satisfy

$$UV = e^{-2\pi i/N} VU = \omega VU \tag{4.1}$$

and are therefore related to $\hat{V}$ and $\hat{U}$ via (restoring θ for a moment)

$$\begin{aligned} \hat{U} &= U^{\sqrt{\theta N/2\pi}} \\ \hat{V} &= V^{\sqrt{\theta N/2\pi}} . \end{aligned} \tag{4.2}$$

The propagators are

$$\langle A_{ij} A_{kl} \rangle = \frac{1}{N} \sum_{mn} :U^m V^n :_{ij} : U^{-m} V^{-n} :_{kl} \frac{1}{(m^2+n^2)/N + M^2} . \tag{4.3}$$

V and U can be represented by the $N \times N$ 'clock' and 'shift' matrices (this was also used in [38, 39]):

$$\begin{aligned} U_{i,i+1} &= 1 \qquad i = 1 \ldots N-1 \\ U_{N,1} &= 1 \\ V_{i,i} &= \omega^{i-1} \qquad \omega = e^{-2\pi i/N} . \end{aligned} \tag{4.4}$$

Using these matrices, we can now go back to the definition of the operator representation of our algebra, (2.7), and rewrite it as

$$\begin{aligned} A_{ij} &= \frac{1}{2\pi N} \sum_{mn} :U^m V^n :_{ij} \phi(\tilde{m}, \tilde{n}) \\ &= \frac{1}{2\pi N} \sum_{n} \omega^{\frac{1}{2} n(i+j-2)} \phi(\tilde{j} - \tilde{i}, \tilde{n}) = \frac{1}{\sqrt{2\pi N}} \phi\left(\tilde{j} - \tilde{i}, \frac{(j+i)}{2}\right) , \end{aligned} \tag{4.5}$$

which shows, as promised, that the matrix indices in the operator formalism correspond to the mixed configuration-momentum space. If we change the indexing of the matrix element (i,j) to $(j-i \bmod N, (i+j)/2 \bmod N)$, the first index corresponds to momentum space while the second to configuration space.

Inserting U and V into the propagators (4.3) gives

$$\langle A_{ij} A_{kl} \rangle = \delta_{i-j,l-k} \frac{1}{N} \sum_n \frac{w^{n(j-k)}}{M^2 + ((j-i)^2 + n^2)/N} \,. \tag{4.6}$$

Taking the limit $N \to \infty$ we should also scale i, j, k and l. Explicitly, we take the combinations $x \equiv 2\pi(j-k)/\sqrt{N}$ and $p \equiv (j-i)/\sqrt{N}$ fixed while taking the limit and write $q \equiv (k-l)/\sqrt{N}$ (which must be also fixed). The sum in the propagator becomes (this is just the usual propagator)

$$\frac{1}{N} \delta(p+q) \int_{-\infty}^{\infty} dk \frac{e^{ikx}}{p^2 + M^2 + k^2} = \frac{\delta(p+q)}{N} \frac{e^{-(p^2+M^2)|x|}}{p^2 + M^2} \,. \tag{4.7}$$

Note that an extra factor of $\sqrt{N}$ comes from the δ-function. This expression for the propagator is, of course, the one we get for the mixed space up to the power of $\frac{1}{N}$ that comes from equation (4.5).

One can easily see that the operator formalism, using the 'clock' and 'shift' matrices, corresponds to the mixed-space formalism by considering the kinetic term — $[\hat{x}^\mu, \hat{A}]^2$. This term comprises of a diagonal term from $\hat{x}^2 = i\log V$ corresponding to the momentum coordinate and a non-diagonal term from $\hat{x}^1 = i\log U$ corresponding to the configuration coordinate. This is the same as in the mixed-space representation, as equation (3.1) shows.

5. Feynman diagrams

If one works with the Moyal product representation of the algebra, that is with functions on $\mathbb{R}^d$ and the multiplication rule (2.2), it is possible to define momentum-space Feynman rules, which differ from the commutative ones only in the K-point vertex, which acquires a phase (depending on the *order* of the momenta flowing into the vertex):

$$V(k_1, k_2 \ldots, k_K) = \lambda e^{-\frac{i}{2} \sum_{i<j} k_i \times k_j} \delta \left(\sum k_i \right) . \tag{5.1}$$

Considering various Feynman graphs, it is shown in [8] that in planar graphs these phases add up so that the overall phase depends only on the external momenta, while in non-planar graphs, one is left with phases depending on internal (integrated) momenta. These phases may give rise to IR divergences in non-planar graphs (The UV/IR mixing).

When one works in a mixed configuration space the propagator takes the form (see equation (4.7))

$$\langle\phi(\tilde{p},x)\phi(\tilde{q},y)\rangle = \delta(p+q)\frac{e^{-(p^2+M^2)|x-y|}}{p^2+M^2}\,. \tag{5.2}$$

The interaction vertex (for ϕ^n) takes the form

$$\begin{aligned}\int dx d^n p\delta\left(\sum_{i=1}^{n}p_i\right)\phi(\tilde{p}_1,x)\phi(\tilde{p}_2,x-(p_1+p_2))\phi(\tilde{p}_3,x-(p_1+2p_2+p_3))\cdots\\ \phi\left(\tilde{p}_j,x-\left(p_1+2\sum_{k=2}^{j-1}p_k+p_j\right)\right)\cdots\phi\left(\tilde{p}_n,x-\left(\sum_{k=2}^{n-1}p_k\right)\right).\end{aligned} \tag{5.3}$$

Given a (non) planar diagram in the momentum space representation of a non-commutative theory, it clearly remains such a diagram in the mixed configuration-momentum space. Thus, the (non) planar diagrams of the momentum spaces representations are the (non) planar diagram of the matrix formalism. Considering the Feynman rules for the matrix formalism, it is clear that the phases in the non-planar diagrams (before performing any sums on external momenta) may come only from the propagators (4.3) as the vertices clearly do not carry any phase. For future use, we wish to consider the simpler case where there is no kinetic term in the action. This may be considered as the $\theta\to\infty$ limit of the action (2.13),[1] which reduces to

$$S=\int dt(M^2\,\mathrm{Tr}\,A^2+\lambda\,\mathrm{Tr}\,A^K)\,. \tag{5.4}$$

The free (tree level) propagators for this theory are simply

$$\langle A_{ij}A_{kl}\rangle = \frac{1}{N}\sum_{mn}:U^mV^n:_{ij}:U^{-m}V^{-n}:_{kl}\frac{1}{M^2}=\frac{1}{M^2}\delta_{j,k}\delta_{i,l}\,, \tag{5.5}$$

which correspond to the totally local propagators[2]

$$\langle\phi(\tilde{p},x)\phi(\tilde{q},y)\rangle\sim\delta(p+q)\delta(x-y)\,. \tag{5.6}$$

This corresponds, of course, to a situation where we have no propagating degrees of freedom in the field theory. However, from (5.5) it is clear that the U's and V's and the phases that arise are the same as with the kinetic term. Thus, for any given diagram, the question whether there is a phase mixing between internal and external momenta, does not change when we drop the kinetic term. We will use this observation later when we work without the kinetic term to clarify the relation between non-planar diagrams and IR physics.

[1] Note that for generic values of the external momenta this limit is dominated by planar diagrams. However, the IR divergences we are interested in, appear in special values of the external momenta

[2] By a local propagator we mean here that it allows propagation only from a point to itself.

6. Origin of IR divergences

The question we wish to answer in this section is — where do the divergences of the non-commutative field theory come from, as viewed from the matrix (operator) formulation. We would like to understand the connection between the IR[3] and UV divergences, and to reach a more intuitive explanation for the connection between those IR effects and non-planar diagrams.

A finite dimensional matrix theory has no divergences, as it is a theory on a point. Infinities can only come from taking $N \to \infty$. Indeed, in a regular matrix theory the N dependence is clear and one usually has a $1/N$ expansion of the physical quantities. The matrix theories arising from the non-commutative field theories have a kinetic term which complicates the N dependence. Furthermore, the natural regularization and renormalization procedures of the field theory throw away the leading N behaviour.

To get infra-red divergences in a correlation function one needs two ingredients:

- An infinite volume
- Long range interactions

We will see now how these conditions come about in the non-planar diagrams of a non-commutative field theory.

Let us first make the following observation: The size of the matrices N, which plays the role of a UV cut-off, as all momenta are in the range $-\sqrt{N} < p < \sqrt{N}$, is also the configuration spaces area (at fixed θ), as

$$\text{Area} = \int d^2x \sim \mathrm{Tr}\, 1 = N\,. \tag{6.1}$$

Hence the volume of space is proportional to N. Therefore, any infra-red effect coming from the large volume of space time are cut-off by N. This is one of the reasons for the IR$\leftrightarrow$UV relation, as N plays the role of both IR and UV cut-off's and there cannot be IR divergences if we keep a finite UV cut-off. This, however does not explain why the IR divergences occur only when the theory has UV divergences (see below).

6.1 Long range interaction

We would like to understand why certain non-planar diagrams are those which give rise to the IR effects. First, let us see how long range interactions are produced. To gain insight into this question it turns out to be very useful to work not in momenta space or configuration space but rather in the mixture of both. As explained in

[3]By IR divergences we refer here only to divergences in some correlation functions, which diverge for vanishing external momenta.

the previous sections, in this representation the star product becomes simpler and more intuitive. Indeed, we have seen that the matrix description corresponds to such a representation and the star product is then just matrix multiplication. So we will work with fields that are functions of one configuration and one momentum coordinate. To avoid annoying factors, we redefine the matrix variables

$$A_{ij} \sim \phi(\widetilde{i-j}, i+j)\,. \tag{6.2}$$

To see the difference between an ordinary and a non-commutative theory let us consider the ϕ^4 theory. In an ordinary field theory the interaction vertex is local, $\int d^2x\phi^4(x)$. This is written in a mixed configuration-momenta representation as

$$\int d^4p dx \prod_{i=1}^{4} \phi(\tilde{p_i}, x)\delta\left(\sum_{i=1}^{4} p_i\right), \tag{6.3}$$

where $\int d^4p = \prod_{i=1}^4 \int dp_i$. On the other hand a non-commutative theory will have a non-local interaction of a special form. For the quartic vertex the interaction takes the form (again, there are some factors of 2 from the previous sections)

$$\int d^4pdx\phi(\tilde{p_1}, x)\phi(\tilde{p_2}, x-p_1-p_2)\phi(\tilde{p_3}, x-p_1-2p_2-p_3)$$
$$\phi(\tilde{p_4}, x-p_1-2p_2-2p_3-p_4)\delta\left(\sum_{i=1}^{4} p_i\right). \tag{6.4}$$

The non-commutative theory has a non-local interaction whose non locality in one direction depends on the momenta in the other direction, as was noticed in [34]. We will show that the long range interactions that are needed for an infra-red divergence arise from this non-local interaction, but only for certain non-planar diagrams.

One can already guess how long range interaction may come about. We have a non-local interaction and a regular massive free propagator. Long range interactions require that correlation functions are not exponentially suppressed for large distances (as they are in local massive theories). Such an effect can occur if the non-local interaction "draws" the interaction points close to each other, so while still having a massive propagator, the interaction is not exponentially suppressed. Given that, it seems clear that the non-locality is proportional to the momenta flowing in the vertex, it is thus obvious that large non-locality is related to high momenta.

As a simple example let us consider the two-point function in a non-commutative ϕ^4 theory. In an ordinary theory the first order (one loop) correction to the two point function is proportional to

$$\int dxd^4p\ \phi(\tilde{p_1}, x)\phi(\tilde{p_2}, x)\langle\phi(\tilde{p_3}, x)\phi(\tilde{p_4}, x)\rangle_0\delta\left(\sum_{i=1}^{4} p_i\right), \tag{6.5}$$

where $\langle\ \rangle_0$ is the free propagator and we postpone the contraction with the external ϕ's. Using the fact that the free propagator has a momentum delta-function, we find

$$\int dx dp_1 dp_2 \phi(\tilde{p_1}, x)\phi(\widetilde{-p_1}, x)\langle\phi(\tilde{p_2}, x)\phi(\widetilde{-p_2}, x)\rangle_0 \,. \tag{6.6}$$

This term is, of course, the source for the UV divergences, as the fields interact at the same point. We would like to see now what happens in the non-commutative theory. There are two possible corrections, depending on the contractions we make in the vertex. If one contracts neighbouring fields, the diagram is planar, while it is non-planar for contraction of non-neighbouring fields. For the planar diagram one gets

$$\int d^4p dx \phi(\tilde{p_1}, x)\phi(\tilde{p_2}, x - p_1 - p_2) \times \\ \times\langle\phi(\tilde{p_3}, x - p_1 - 2p_2 - p_3)\phi(\tilde{p_4}, x - p_1 - 2p_2 - 2p_3 - p_4)\rangle_0 \delta\left(\sum_{i=1}^{4} p_i\right).$$

The free propagator is still proportional to a momentum delta-function and the correlation function reduces to

$$\int dx dp_1 dp_3 \phi(\tilde{p_1}, x)\phi(-\tilde{p_1}, x)\langle\phi(\tilde{p_3}, x + p_1 - p_3)\phi(-\tilde{p_3}, x + p_1 - p_3)\rangle_0 \,.$$

This is basically the same as the ordinary interaction coming from the local vertex. On the other hand, the non-planar diagram gives

$$\int d^4p dx \phi(\tilde{p_1}, x)\phi(\tilde{p_3}, x - p_1 - 2p_2 - p_3) \times \\ \times\langle\phi(\tilde{p_2}, x - p_1 - p_2)\phi(\tilde{p_4}, x - p_1 - 2p_2 - 2p_3 - p_4)\rangle_0 \delta\left(\sum_{i=1}^{4} p_i\right).$$

Using momentum conservation we get now

$$\int dp_1 dp_2 dx \phi(\tilde{p_1}, x)\phi(-\tilde{p_1}, x - 2p_2)\langle\phi(\tilde{p_2}, x - p_1 - p_2)\phi(-\tilde{p_2}, x + p_1 - p_2)\rangle_0 \,.$$

We can see here that the non-planar diagram exhibits a different behaviour. The non-planar contraction of two 'inner-legs' of the vertex, has caused the two 'outer-legs' to be in different points in configuration space. These points are further away the larger the momentum in the loop is. Thus, the long range forces are related to high momentum. We also see that the loop integral will not diverge unless $p_1 = 0$ as the inner propagator is suppressed for any other value.

Let us look at another example, that of a cubic theory. The interaction vertex is

$$\int d^3p dx \phi(\tilde{p_1}, x)\phi(\tilde{p_2}, x - p_1 - p_2)\phi(\tilde{p_3}, x - p_1 - 2p_2 - p_3)\delta\left(\sum_{i=1}^{3} p_i\right). \tag{6.7}$$

Consider, again, the one loop correction to the two point function. The planar diagram contribution is

$$\int d^3pd^3qdxdy\phi(\tilde{p}_1,x)\phi(\tilde{q}_1,y)\langle\phi(\tilde{p}_2,x-p_1-p_2)\phi(\tilde{q}_3,y-q_2)\rangle_0 \times$$
$$\times\langle\phi(\tilde{p}_3,x-p_2)\phi(\tilde{q}_2,y-q_1-q_2)\rangle_0\delta\left(\sum_{i=1}^{3}p_i\right)\delta\left(\sum_{i=1}^{3}q_i\right). \quad (6.8)$$

Using the various momentum delta-functions, we get

$$\int dp_1dp_2dxdy\phi(\tilde{p}_1,x)\phi(-\tilde{p}_1,y)\langle\phi(\tilde{p}_2,x-p_1-p_2)\phi(-\tilde{p}_2,y-p_1-p_2)\rangle_0 \times$$
$$\times\langle\phi(-(\tilde{p}_1+\tilde{p}_2),x-p_2)\phi(\tilde{p}_1+\tilde{p}_2,y-p_2)\rangle_0 .$$

All propagators are ordinary free propagators and we see that they are evaluated at separation $(x-y)$ (although the points are shifted the propagator is a function of the difference of the points which remains unchanged). Thus, for $x \neq y$ the contribution from the propagators is exponentially suppressed and there is no non-locality. In fact, this looks like an ordinary cubic one-loop correction

On the other hand, the non-planar diagram contribution is

$$\int dp_1dp_2dxdy\phi(\tilde{p}_1,x)\phi(-\tilde{p}_1,y)\langle\phi(\tilde{p}_2,x-p_1-p_2)\phi(-\tilde{p}_2,y+p_1+p_2)\rangle_0 \times$$
$$\times\langle\phi(-(\tilde{p}_1+\tilde{p}_2),x-p_2)\phi(\tilde{p}_1+\tilde{p}_2,y+p_2)\rangle_0 .$$

Again, we see here a different behaviour. First, the inner propagators are evaluated at configuration space distances $x-y-2p_1-2p_2$ and $x-y-2p_2$ respectively. Thus, even for $x \neq y$ the diagram is not always suppressed by the massive propagator. One can also see why the divergences occur in the infra red, as the distances are equal only when $p_1 = 0$. At any other case, at least one of the propagator is suppressed. The same remarks as in the quartic coupling apply here.

To summarize, as the external legs in a non-planar diagram are located further apart, the main contribution is due to higher and higher momenta. Thus, integrating over all configuration space, the large contributions come from large momenta and it would look as if the infra-red effects are connected to UV physics. Another point to notice is that, as the divergences of a diagram come from the high momentum contribution of the loop (as the IR divergences come from the summing over larger and larger separations), there will be an IR divergence only if this loop diverges and there are also UV divergences. This is because the only source of divergences in perturbation theory is that of coincident points. If there are no UV divergences there might still be non-local effects, but there will not be long range interactions, although there will be interaction on a larger scale than the same local theory. We have thus described the mechanism through which long range interactions appear in non-commutative field theories.

6.2 Why non-planar?

We would like now to address the question of the relation between non-planar diagrams and the IR physics from another point of view. For that purpose, we return to the $\theta \to \infty$ limit where there is no kinetic term in the action.

We claim that the essential point for our discussion — the appearance of IR divergences — does not depend on the inclusion of the kinetic term. There are several arguments in favour of this claim. As explained in [8], the IR divergences appear due to the phases depending on the internal and external momenta. It was demonstrated in section 5 that these phases indeed do not depend on the inclusion of the kinetic term. Furthermore, if we choose the propagator to be totally local, the only non-local effects in any correlation function are those coming from the non-local interactions (rather than from any non-locality in the propagator), thus the non-local effects we are after are essentially the same, and even clearer, once the kinetic term is neglected. Finally, the propagator (4.7) of the full theory, with the kinetic term, is peaked around the values of the local (5.6) and is exponentially suppressed away from these values.

Let us consider a general n-point correlation function in the theory without the kinetic term.

$$\langle A_{i_1,j_1} A_{i_2,j_2} A_{i_3,j_3} \cdots A_{i_n,j_n} \rangle \,. \tag{6.9}$$

The propagators (5.5) show us that we can use the two line notation with a matrix index on each line. A general planar diagram, depicted in figure 1, contributes to the correlation function (6.9) only for the index structure

$$j_m = i_{m+1} \bmod n \,. \tag{6.10}$$

A non-planar diagram, on the other hand, may be of a more general index structure.

We see here an important difference between planar and non-planar diagrams — in a planar diagram there is only one group of indices that is connected in a cyclic manner, while a non-planar diagram can contribute even if the outside indices are separated into two or more groups. For example, the separation into two groups gives

$$\begin{aligned} j_m &= i_{m+1} \qquad m = 1 \cdots n_1 - 1 \,, \qquad i_1 = j_{n_1 - 1} \,. \\ j_m &= i_{m+1} \qquad m = n_1 + 1 \cdots n - 1 \,, \qquad i_{n_1+1} = j_n \,. \end{aligned} \tag{6.11}$$

Figure 2 is an example of such a diagram. We will now show that the structure of the phases is in one to one correspondence with the allowed index structure.

Let us look at a contribution to the correlation function (6.9). We will label the incoming momenta by (m_l, n_l) and the inside momenta by (p_k, q_k). The contribution from any diagram is then

$$\sum_{m_l,n_l,p_k,q_k} \left(\prod_h : U^{m_h} V^{n_h} :_{i_h,j_h} \right) \tilde{f}(m_l, n_l . p_k, q_k) \,. \tag{6.12}$$

JHEP08(2001)001

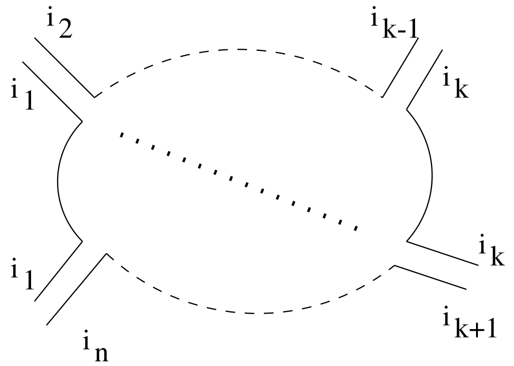

Figure 1: A general planar diagram.

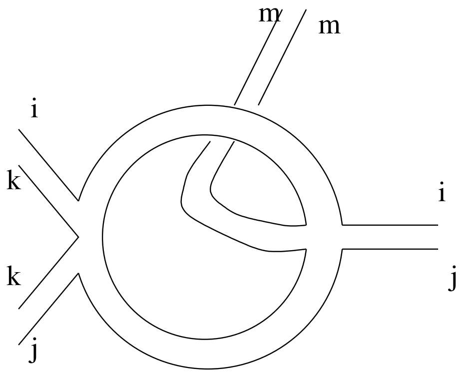

Figure 2: A non planar ϕ^4 4-points diagram.

Given the simple propagator and vertex, the function $\tilde{f}$ can only be made out of some delta functions of momenta and phases. If the phases do not mix between internal and external momenta then after summing over the internal momenta one gets only

$$f(m_l, n_l) \equiv \sum_{p_k, q_k} \tilde{f}(m_l, n_l, p_k, q_K) = cN^a e^{i\alpha(m_l,n_l)} \delta_{(\sum_l m_l, 0)} \delta_{(\sum_l n_l, 0)} , \tag{6.13}$$

where $\alpha(m_l, n_l)$ is some function of the external momenta and c and a are constants. If, however, there is some mixing between the internal and external momenta then after summing over the internal momenta one gets extra delta functions

$$f(m_l, n_l) = c' N^{a'} e^{i\alpha'(m_l,n_l)} \delta_{(\sum_l m_l, 0)} \delta_{(\sum_l n_l, 0)} \cdots \delta_{(\sum_l' m_l, 0)} \delta_{(\sum_l' n_l, 0)} , \tag{6.14}$$

where the primed sums indicate that the sum is not over all l. We see that the structure of delta functions in $f(m_l, n_l)$ is determined by the mixing (or non mixing) of the phases.

On the other hand, $f(m_l, n_l)$ is also determined by the allowed index structure, as one can just do an inverse transform by multiplying by

$$\prod_{h'} : U^{m'_{h'}} V^{n'_{h'}} :_{i_{h'}, j_{h'}} , \tag{6.15}$$

and performing a trace. This shows that there must be a one-to-one correspondence between the index structure and the phase mixing. As an example let us first look at a planar diagram. The index structure is always as in equation (6.10), so one finds

$$f(m_l, n_l) \sim \mathrm{Tr}\left(\prod_h : U^{m_h} V^{n_h} : \right) \sim \delta_{(\sum_l m_l, 0)} \delta_{(\sum_l n_l, 0)} . \tag{6.16}$$

On the other hand a non-planar diagram with an index structure as in equation (6.11) will give

$$\begin{aligned} f(m_l, n_l) \sim\ & \mathrm{Tr}\left(\prod_{h=1}^{h_1} : U^{m_h} V^{n_h} : \right) \mathrm{Tr}\left(\prod_{h=h_1+1}^{n} : U^{m_h} V^{n_h} : \right) \\ & \sim \delta_{(\sum_l m_l, 0)} \delta_{(\sum_l n_l, 0)} \delta_{(\sum_l^{h_1} m_l, 0)} \delta_{(\sum_l^{h_1} n_l, 0)} . \end{aligned} \tag{6.17}$$

We have thus shown that phase mixing between internal and external momenta is equivalent to the grouping of the external indices. As a result, the phase mixing can only occur for non-planar diagrams with a particular index structure.

Using the relation of the matrix indices to the mixed momentum — configuration space, we interpret this difference as follows. In any n-point planar diagram, given the configuration space coordinates of $(n-1)$ points, the coordinate of the last point is fixed. IR effects are related to non-planar diagrams, because in some non-planar diagrams some of the legs can be taken to be independently far from the others (even if the propagator is totally local). The freedom in the index structure of non-planar diagram, given the local form of the propagator, explains why the non-local effects discussed previously occur only for the non-planar diagrams.

As we claimed before that the classification of diagrams to planar and non-planar in the momentum space representation of non-commutative theories is the same as in the matrix formalism, it is clear now why non-planar diagrams are connected to IR effects.

Let us consider the simplest example of this behaviour — the ϕ^4 two point function. Planar diagrams contribute only to (there is no summation on the indices)

$$\langle A_{ij} A_{ji} \rangle \,, \tag{6.18}$$

while non-planar can also contribute to

$$\langle A_{ii} A_{jj} \rangle \,. \tag{6.19}$$

Thus, even though we started with a totally localized propagator (i.e proportional to $\delta(x-y)$) the non-planar graphs may lead to a non-local correction, while the planar graphs may not. We see that this behaviour is totally encoded in the index structure of the possible planar and non-planar diagrams. This is not to say that non-planar diagrams have to be of this form, but that only non-planar diagrams that separate into two or more groupings of indices, are those that will exhibit the IR divergences.

Diagrams with the same index structure as planar ones will have no phase mixing and will not exhibit IR divergences related to the values of the outside momenta.

6.3 The power of N

One may still be interested in the degree of the IR divergences (or — how do the non-planar diagram end up with the same power of N as planar ones). The degree of divergence in the theory without the kinetic term, is simply the N dependences. To consider that, let us look, once again, at the example of the two point function in ϕ^4 theory. We will look at the contributions to the two-point function at zero momenta, $\langle \tilde{A}(0,0)\tilde{A}(0,0)\rangle$ where $\tilde{A}(0,0) = \mathrm{Tr}\, A$. The planar diagram contributes (at one loop) to terms of the form

$$\langle A_{ij} A_{ji} \rangle \sim \frac{1}{M^2} N \delta_{i,j} \,, \tag{6.20}$$

JHEP08(2001)001

while the non-planar diagram contributes to terms of the form

$$\langle A_{ii} A_{jj} \rangle \sim \frac{1}{M^2}\,, \tag{6.21}$$

where there is no summation in both cases. We see that the planar diagrams have a higher N dependence than the non-planar ones. However, there are N planar terms that contribute to this two-point function, but N^2 non-planar terms, thus giving and overall identical N dependence.

We can generalize this picture to any n-point function. Transforming the planar diagram from the matrix space to momentum space we have a single trace over $\hat{U}$'s and $\hat{V}$'s:

$$\begin{aligned}\langle \tilde{\hat{A}}(p_1, q_1) \ldots \tilde{\hat{A}}(p_n, q_n) \rangle &\sim \mathrm{Tr}(\hat{U}^{p_1} \hat{V}^{q_1} \ldots \hat{U}^{p_n} \hat{V}^{q_n}) \cdots \\ &= N \cdot \delta_{p_1+\cdots+p_n,0} \delta_{q_1+\cdots+q_n,0} \times \text{phase} \cdots\,,\end{aligned} \tag{6.22}$$

where the δ corresponds to the usual momentum conservation δ-function. A non-planar diagram, which might have a different grouping of the external indices, will give

$$\begin{aligned}&\langle \tilde{\hat{A}}(p_1, q_1) \ldots \tilde{\hat{A}}(p_k, q_k) \tilde{\hat{A}}(p_{k+1} q_{k+1}) \ldots \tilde{\hat{A}}(p_n, q_n) \rangle \sim \\ &\qquad \sim \mathrm{Tr}(\hat{U}^{p_1} \hat{V}^{q_1} \ldots \hat{U}^{p_k} \hat{V}^{q_k}) \times \mathrm{Tr}(\hat{U}^{p_{k+1}} \hat{V}^{q_{k+1}} \ldots \hat{U}^{p_n} \hat{V}^{q_n}) \ldots \\ &\qquad = N^2 \cdot \delta_{p_1+\cdots+p_k,0} \delta_{q_1+\cdots+q_k,0} \delta_{p_{k+1}+\cdots+p_n,0} \delta_{q_{k+1}+\cdots+q_n,0} \times \text{phase} \cdots\,.\end{aligned} \tag{6.23}$$

The additional traces which give additional factors of N as well as new kinematical restrictions are the hallmark of the possible IR divergences. The corresponding factors of N may cause these diagrams to be of the same order in N as the planar diagrams.

The various factors of N are also related to a fact briefly mentioned before. As mentioned for example in [8], the $\theta \to \infty$ limit is dominated at generic external momenta by the planar diagrams. This is obvious from equations (6.20) and (6.21). However, at special values of the external momenta, this dominance may be compensated by the additional traces and we may encounter non-planar diagrams which contribute leading (and even diverging) terms.

A. A simple example

As a demonstration of the origin of the phases in the operators formalism, we give a simple example — consider the one loop, two point function in non-commutative ϕ^3. The expectation value to be computed is

$$\langle \hat{A}_{ab} \hat{A}_{cd} \,\mathrm{Tr}\, \hat{A}^3 \,\mathrm{Tr}\, \hat{A}^3 \rangle = \langle \hat{A}_{ab} \hat{A}_{cd} \hat{A}_{a_1 b_1} \hat{A}_{b_1 c_1} \hat{A}_{c_1 a_1} \hat{A}_{a_2 b_2} \hat{A}_{b_2 c_2} \hat{A}_{c_2 a_2} \rangle\,. \tag{A.1}$$

(a) Planar graph (b) Non-planar graph

Figure 3: one-loop Feynman graphs.

There are two distinct Feynman graphs corresponding to this expression, a planar graph and a non planar one (see figure 3).

$$
\begin{aligned}
&\langle \hat{A}_{ab}\hat{A}_{a_1b_1}\rangle\langle \hat{A}_{b_1c_1}\hat{A}_{a_2b_2}\rangle\langle \hat{A}_{c_1a_1}\hat{A}_{c_2a_2}\rangle\langle \hat{A}_{b_2c_2}\hat{A}_{cd}\rangle = \\
&= \sum_{m_in_i} :\hat{U}^{m_1}\hat{V}^{n_1}:_{ab}:\hat{U}^{-m_1}\hat{V}^{-n_1}:_{a_1b_1}:\hat{U}^{m_2}\hat{V}^{n_2}:_{b_1c_1}:\hat{U}^{-m_2}\hat{V}^{-n_2}:_{a_2b_2} \\
&\qquad :\hat{U}^{m_3}\hat{V}^{n_3}:_{c_1a_1}:\hat{U}^{-m_3}\hat{V}^{-n_3}:_{c_2a_2}:\hat{U}^{m_4}\hat{V}^{n_4}:_{b_2c_2}:\hat{U}^{-m_4}\hat{V}^{-n_4}:_{cd}\ \ldots \\
&= \sum_{m_in_i} :\hat{U}^{m_1}\hat{V}^{n_1}:_{ab}:\hat{U}^{-m_4}\hat{V}^{-n_4}:_{cd}\ \mathrm{Tr}\, fe\left(:\hat{U}^{-m_1}\hat{V}^{-n_1}::\hat{U}^{m_2}\hat{V}^{n_2}::\hat{U}^{m_3}\hat{V}^{n_3}:\right) \\
&\qquad \mathrm{Tr}\left(:\hat{U}^{-m_2}\hat{V}^{-n_2}::\hat{U}^{m_4}\hat{V}^{n_4}::\hat{U}^{-m_3}\hat{V}^{-n_3}:\right)\ldots \\
&= \sum_{m_in_i} :\hat{U}^{m_1}\hat{V}^{n_1}:_{ab}:\hat{U}^{-m_4}\hat{V}^{-n_4}:_{cd} \times \exp(i\theta(m_1n_1/2 + m_2n_2 + m_3n_3 + \\
&\qquad + m_4n_4/2 - m_2n_1 - m_3n_1 - m_4n_2 - m_3n_4 + 2m_3n_2)) \times \\
&\qquad \times \delta(m_1 - m_2 - m_3)\delta(m_4 - m_2 - m_3)\delta(n_1 - n_2 - n_3)\delta(n_4 - n_2 - n_3)\ldots ,
\end{aligned}
$$

where $\ldots$ represent further terms that do not contribute to the phase. It can be easily seen that the phase in the last line adds up to zero and one is left only with the phase coming from the external legs.

The planar graph is given by the contraction.

The non-planar graph is given by

$$
\begin{aligned}
&\langle \hat{A}_{ab}\hat{A}_{a_1b_1}\rangle\langle \hat{A}_{b_1c_1}\hat{A}_{a_2b_2}\rangle\langle \hat{A}_{c_1a_1}\hat{A}_{b_2c_2}\rangle\langle \hat{A}_{c_2a_2}\hat{A}_{cd}\rangle = \\
&= \sum_{m_in_i} :\hat{U}^{m_1}\hat{V}^{n_1}:_{ab}:\hat{U}^{-m_1}\hat{V}^{-n_1}:_{a_1b_1}:\hat{U}^{m_2}\hat{V}^{n_2}:_{b_1c_1}:\hat{U}^{-m_2}\hat{V}^{-n_2}:_{a_2b_2} \\
&\qquad :\hat{U}^{m_3}\hat{V}^{n_3}:_{c_1a_1}:\hat{U}^{-m_3}\hat{V}^{-n_3}:_{b_2c_2}:\hat{U}^{m_4}\hat{V}^{n_4}:_{c_2a_2}:\hat{U}^{-m_4}\hat{V}^{-n_4}:_{cd}\ \ldots \\
&= \sum_{m_in_i} :\hat{U}^{m_1}\hat{V}^{n_1}:_{ab}:\hat{U}^{-m_4}\hat{V}^{-n_4}:_{cd}\ \mathrm{Tr}\left(:\hat{U}^{-m_1}\hat{V}^{-n_1}::\hat{U}^{m_2}\hat{V}^{n_2}::\hat{U}^{m_3}\hat{V}^{n_3}:\right) \\
&\qquad \mathrm{Tr}\left(:\hat{U}^{-m_2}\hat{V}^{-n_2}::\hat{U}^{-m_3}\hat{V}^{-n_3}::\hat{U}^{m_4}\hat{V}^{n_4}:\right)\ldots
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{m_i n_i} :\hat{U}^{m_1}\hat{V}^{n_1}:_{ab}: \hat{U}^{-m_4}\hat{V}^{-n_4}:_{cd} \times \exp(i\theta(m_1 n_1/2 + m_2 n_2 + m_3 n_3 + \\
&\quad + m_4 n_4/2 - m_2 n_1 - m_3 n_1 - m_4 n_3 - m_4 n_2 + 2 m_3 n_2)) \times \\
&\quad \times \delta(m_1 - m_2 - m_3)\delta(m_4 - m_2 - m_3)\delta(n_1 - n_2 - n_3)\delta(n_4 - n_2 - n_3)\dots \\
&= \sum_{m_i n_i} :\hat{U}^{m_1}\hat{V}^{n_1}:_{ab}: \hat{U}^{-m_4}\hat{V}^{-n_4}:_{cd} \exp\left(i\theta(-m_2 n_3 + m_3 n_2)\right)\dots
\end{aligned}
$$

giving the same phase as the corresponding diagram in momentum space.

References

[1] A. Connes, *Noncommutative geometry*, Academic Press 1994.

[2] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162].

[3] M.R. Douglas and C. Hull, *D-branes and the noncommutative torus*, *J. High Energy Phys.* **02** (1998) 008 [hep-th/9711165].

[4] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[5] M.M. Sheikh-Jabbari, *Super Yang-Mills theory on noncommutative torus from open strings interactions*, *Phys. Lett.* **B 450** (1999) 119 [hep-th/9810179].

[6] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

[7] N. Seiberg, *A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation*, *J. High Energy Phys.* **09** (2000) 003 [hep-th/0008013].

[8] S. Minwalla, M. Van Raamsdonk and N. Seiberg, *Noncommutative perturbative dynamics*, *J. High Energy Phys.* **02** (2000) 020 [hep-th/9912072].

[9] M. Van Raamsdonk and N. Seiberg, *comments on noncommutative perturbative dynamics*, *J. High Energy Phys.* **03** (2000) 035, [hep-th/0002186].

[10] I.Y. Aref'eva, D.M. Belov and A.S. Koshelev, *Two-loop diagrams in noncommutative Φ_4^4 theory*, *Phys. Lett.* **B 476** (2000) 431 [hep-th/9912075].

[11] I.Y. Aref'eva, D.M. Belov and A.S. Koshelev, *A note on UV/IR for noncommutative complex scalar field*, hep-th/0001215.

[12] I.Y. Aref'eva, D.M. Belov, A.S. Koshelev and O.A. Rytchkov, *UV/IR mixing for noncommutative complex scalar field theory, 2. Interaction with gauge fields*, *Nucl. Phys.* **102** *(Proc. Suppl.)* (2001) 11 [hep-th/0003176].

[13] A. Micu, *Noncommutative Φ^4 theory at two loops*, *J. High Energy Phys.* **01** (2001) 025 [hep-th/0008057].

[14] W.-H. Huang, *Two-loop effective potential in noncommutative scalar field theory*, *Phys. Lett.* **B 496** (2000) 206 [hep-th/0009067].

[15] F. Zamora, *On the operator product expansion in noncommutative quantum field theory*, *J. High Energy Phys.* **05** (2000) 002 [hep-th/0004085].

[16] S. Imai and N. Sasakura, *Scalar field theories in a Lorentz-invariant three-dimensional noncommutative space-time*, *J. High Energy Phys.* **09** (2000) 032 [hep-th/0005178].

[17] J. Gomis, K. Landsteiner and E. Lopez, *Non-relativistic non-commutative field theory and UV/IR mixing*, *Phys. Rev.* **D 62** (2000) 105006 [hep-th/0004115].

[18] A. Matusis, L. Susskind and N. Toumbas, *The IR/UV connection in the non-commutative gauge theories*, *J. High Energy Phys.* **12** (2000) 002 [hep-th/0002075].

[19] O. Andreev and H. Dorn, *Diagrams of noncommutative Φ^3 theory from string theory*, *Nucl. Phys.* **B 583** (2000) 145 [hep-th/0003113].

[20] Y. Kiem and S. Lee, *Uv/ir mixing in noncommutative field theory via open string loops*, *Nucl. Phys.* **B 586** (2000) 303 [hep-th/0003145].

[21] A. Rajaraman and M. Rozali, *Noncommutative gauge theory, divergences and closed strings*, *J. High Energy Phys.* **04** (2000) 033 [hep-th/0003227].

[22] A. Armoni, *Comments on perturbative dynamics of non-commutative yang- mills theory*, *Nucl. Phys.* **B 593** (2001) 229 [hep-th/0005208].

[23] A. Armoni, R. Minasian and S. Theisen, *On non-commutative $N = 2$ super Yang-Mills*, *Phys. Lett.* **B 513** (2001) 406 [hep-th/0102007].

[24] Y. Kiem, S. Lee and J. Park, *Noncommutative field theory from string theory: two-loop analysis*, *Nucl. Phys.* **B 594** (2001) 169 [hep-th/0008002].

[25] I. Chepelev and R. Roiban, *Renormalization of quantum field theories on noncommutative R^d, 1. Scalars*, *J. High Energy Phys.* **05** (2000) 037 [hep-th/9911098].

[26] I. Chepelev and R. Roiban, *Convergence theorem for non-commutative feynman graphs and renormalization*, *J. High Energy Phys.* **03** (2001) 001 [hep-th/0008090].

[27] H. Grosse, T. Krajewski and R. Wulkenhaar, *Renormalization of noncommutative Yang-Mills theories: a simple example*, hep-th/0001182.

[28] L. Bonora and M. Salizzoni, *Renormalization of noncommutative* U(N) *gauge theories*, *Phys. Lett.* **B 504** (2001) 80 [hep-th/0011088].

[29] A. Bichl, J. Grimstrup, H. Grosse, L. Popp, M. Schweda and R. Wulkenhaar, *Renormalization of the noncommutative photon selfenergy to all orders via Seiberg-Witten map*, *J. High Energy Phys.* **06** (2001) 013 [hep-th/0104097].

[30] S. Sarkar and B. Sathiapalan, *Comments on the renormalizability of the broken phase in noncommutative scalar field theory*, *J. High Energy Phys.* **05** (2001) 049 [hep-th/0104106].

[31] L. Griguolo and M. Pietroni, *Hard non-commutative loops resummation*, hep-th/0102070.

[32] L. Griguolo and M. Pietroni, *Wilsonian renormalization group and the non-commutative IR/UV connection*, *J. High Energy Phys.* **05** (2001) 032 [hep-th/0104217].

[33] V.V. Khoze and G. Travaglini, *Wilsonian effective actions and the IR/UV mixing in noncommutative gauge theories*, *J. High Energy Phys.* **01** (2001) 026 [hep-th/0011218].

[34] D. Bigatti and L. Susskind, *Magnetic fields, branes and noncommutative geometry*, *Phys. Rev.* **D 62** (2000) 066004 [hep-th/9908056].

[35] R. Gopakumar, S. Minwalla and A. Strominger, *Noncommutative solitons*, *J. High Energy Phys.* **05** (2000) 020 [hep-th/0003160].

[36] S. Iso, H. Kawai and Y. Kitazawa, *Bi-local fields in noncommutative field theory*, *Nucl. Phys.* **B 576** (2000) 375 [hep-th/0001027].

[37] D.J. Gross and N.A. Nekrasov, *Solitons in noncommutative gauge theory*, *J. High Energy Phys.* **03** (2001) 044 [hep-th/0010090].

[38] G. Landi, F. Lizzi and R.J. Szabo, *From large-N matrices to the noncommutative torus*, *Comm. Math. Phys.* **217** (2001) 181 [hep-th/9912130].

[39] P. Kraus, A. Rajaraman and S. Shenker, *Tachyon condensation in noncommutative gauge theory*, *Nucl. Phys.* **B 598** (2001) 169 [hep-th/0010016].